Mathematics of Symmetry: An Experimental Core Mathematics Course

by

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1. Summary and Goals of the Project

The project was to develop and teach a course on the mathematics of symmetry to students majoring in the liberal arts, communications and fine arts, and film and television, as an alternative to Math 102 (Quantitative Methods for the Modern World). The first goal of the course was to convince students that mathematics was not just about calculation, formulas and procedures, but also about identifying and generalizing patterns, critical thinking, classification and proof. The second goal was to convince them that mathematics had connections to the arts and humanities, as well as the sciences.

2. Description of the Course and Data

The course was based on the book *Groups and Symmetry: A Guide to Discovering Mathematics*, by David Farmer. The course discussed the rigid motions of the plane (translations, rotations, reflections and glide reflections), the mathematical definition of a symmetry of a two-dimensional figure as a rigid motion of the plane which leaves the figure unchanged, and the recognition and classification of the symmetries of finite figures, one-dimensional strip patterns, and two-dimensional wallpaper patterns. The approach taken by the text and the course was discovery based – students primarily worked through a series of tasks which developed the ideas and explored examples, with relatively little lecturing. Two exams tested the students mastery of this content.

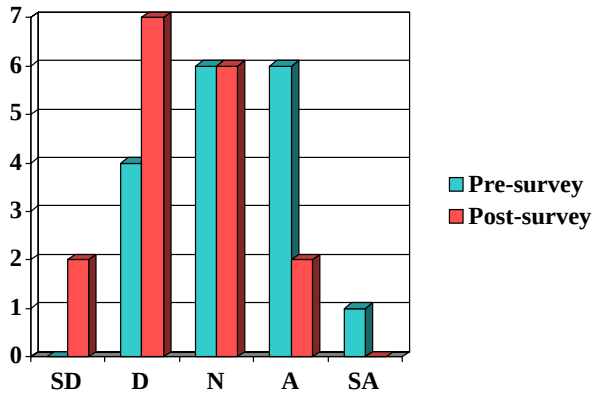
Students were also given projects to explore the connections between symmetry and the arts or other areas. Most notably, each student completed an individual or group project on a topic of their choice, wrote a paper, and made a presentation to the class.

Both quantitative and qualitative data were collected. At the beginning and end of the course, students were asked to complete an anonymous on-line survey in which they indicated their level of agreement or disagreement with 36 statements, using a Likert scale from Strongly Agree through Strongly Disagree. The survey also contained several short free-response questions, including “What is mathematics?”. In addition, copies were made of all assignments turned in, including homework, exams, and projects.

3. Results of the Project and Analysis of the Data

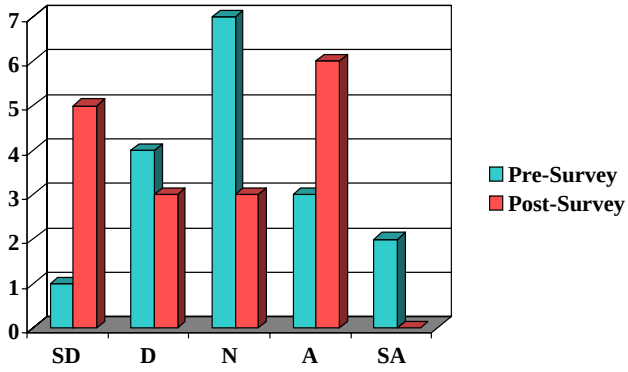
The survey was taken by 31 students at the beginning of the semester and 21 students at the end. Of these, we were able to match 17 sets of responses at the beginning and end. The survey questions fell into four broad categories, relating to the students beliefs about the nature of mathematics, their interest in mathematics, their view of the usefulness of mathematics, and their confidence in doing mathematics. Several of these questions showed interesting results. To illustrate, we will look at one question from each of the four broad categories. In the charts below we compare the responses for the 17 matched students at the beginning and end of the course.

(i) Beliefs about mathematics: *Mathematics is mostly facts and procedures that have to be memorized.*



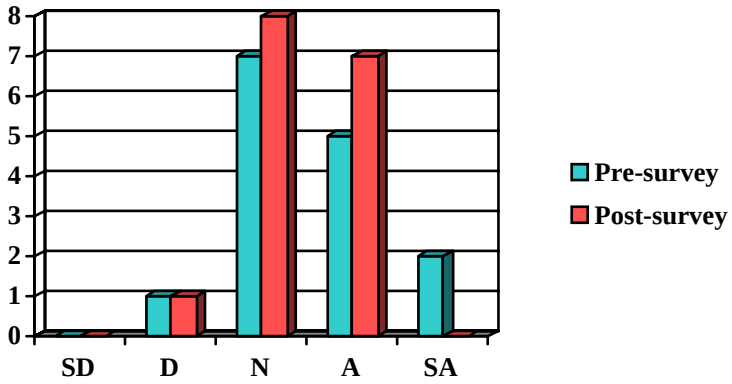
As desired, fewer students agreed with this statement at the end of the course than at the beginning.

(ii) Interest in mathematics: *I am interested in learning more mathematics.*



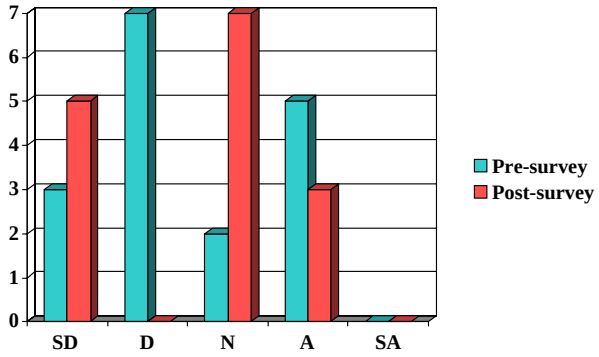
Here, it is interesting to note the bifurcation among the students, with more students both agreeing and disagreeing, but fewer remaining neutral.

(iii) Usefulness of mathematics: *Mathematical thinking is useful in the humanities.*



There was very little change in the response to this question.

(iv) Confidence in doing mathematics: *I have a lot of self-confidence when it comes to mathematics*.



As in the question on interest in mathematics, these reponses showed a split between the students, with several gaining confidence, and others losing it.

Overall, the clearest results were in the questions pertaining to students' beliefs about the nature of mathematics – the results in the other areas were mixed or minimal. Students' comments about the course reinforce this conclusion, with some students calling it the most interesting math course they had ever taken, and others saying they were very disappointed.

The final projects, on topics ranging from Islamic textiles to bodybuilding to *2001: A Space Odyssey*, were generally well done and very interesting. Students made thoughtful explorations of the appearance of symmetry in a range of contexts.

4. Conclusions and plans for the future

While the results of the course were mixed, it was certainly worthwhile, and we plan to do some version of it again. Students did, overall, broaden their view of mathematics to include the analysis of visual patterns, in addition to the manipulation of numbers and algebraic equations. Some students saw more connections between mathematics and the arts and humanities, though not as many (or as strongly) as we had hoped. The mathematical idea of a proof was not understood by most of the students, and the ability to do critical thinking in mathematics was mixed.

The projects were very successful, but were too weighted towards the end of the semester. In the future, it would be better to integrate connections to the arts and humanities throughout the course, and perhaps even team-teach the course with a faculty member in the arts. The material of the course should be reorganized to spread out the analysis of rigid motions and begin looking at actual figures and designs more quickly. Along the way, the notion of proof needs to be addressed more explicitly and directly.

5. Dissemination

Results from this project were presented at the 3rd Annual Conference of the International Society for the Scholarship of Teaching and Learning in Washington D.C. in November, 2006, and at the Workshop on Innovations in Mathematics Education via the Arts at the Banff International Research Station in January, 2007. The results will also be presented at the MathFest conference in San Jose in August, 2007. We hope to write up the results in an article, possibly after having the chance to teach the course again.